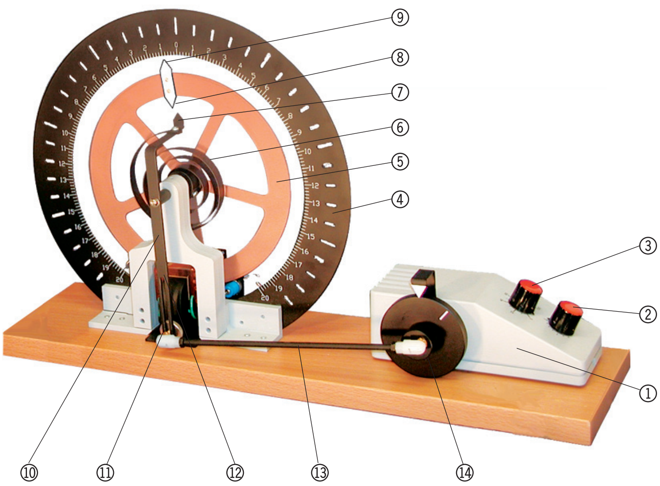
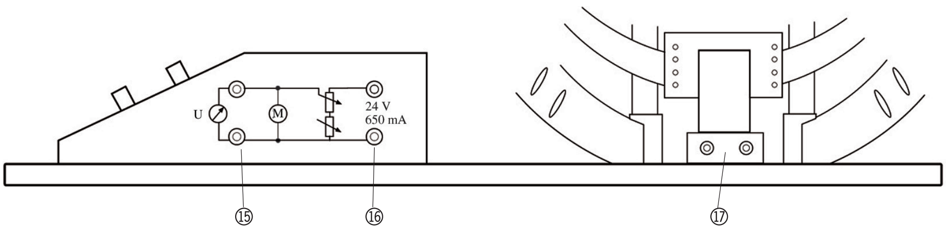
U15040 Torsion pendulum according to Professor Pohl

Operating instructions

12/03 ALF



1. Exciter motor
2. Control knob for fine adjustment of the exciter voltage
3. Control knob for coarse adjustment of the exciter voltage
4. Scale ring
5. Pendulum body
6. Coil spring
7. Pointer for the exciter phase angle
8. Pointer for the pendulum's phase angle
9. Pointer for the pendulum's deflection
10. Exciter
11. Eddy current brake
12. Guide slot and screw to set the exciter amplitude
13. Connecting rod
14. Eccentric drive wheel
15. 4-mm safety socket for exciter voltage measurement
16. 4-mm safety sockets for the exciter motor power supply
17. 4-mm safety sockets for the eddy current brake power supply



The torsion pendulum may be used to investigate free, forced and chaotic oscillations with various degrees of damping.

Experiment topics:

- Free rotary oscillations at various degrees of damping (oscillations with light damping, aperiodic oscillation and aperiodic limit)
- Forced rotary oscillations and their resonance curves at various degrees of damping
- Phase displacement between the exciter and resonator during resonance
- Chaotic rotary oscillations
- Static determination of the direction variable D
- Dynamic determination of the moment of inertia J

1. Safety instructions

- When removing the torsional pendulum from the packaging do not touch the scale ring. This could lead to damage. Always remove using the handles provided in the internal packaging.
- When carrying the torsional pendulum always hold it by the base plate.
- Never exceed the maximum permissible supply voltage for the exciter motor (24 V DC).
- Do not subject the torsional pendulum to any unnecessary mechanical stress.

2. Description, technical data

The Professor Pohl torsional pendulum consists of a wooden base plate with an oscillating system and an electric motor mounted on top. The oscillating system is a ball-bearing mounted copper wheel (5), which is connected to the exciter rod via a coil spring (6) that provides the restoring torque. A DC motor with coarse and fine speed adjustment is used to excite the torsional pendulum. Excitement is brought about via an eccentric wheel (14) with connecting rod (13) which

unwinds the coil spring then compresses it again in a periodic sequence and thereby initiates the oscillation of the copper wheel. The electromagnetic eddy current brake (11) is used for damping. A scale ring (4) with slots and a scale in 2-mm divisions extends over the outside of the oscillating system; indicators are located on the exciter and resonator.
The device can also be used in shadow projection demonstrations.
A DC power supply unit for the torsional pendulum U11755 is required to power the equipment.

Natural frequency:	0.5 Hz approx.
Exciter frequency:	0 to 1.3 Hz (continuously adjustable)
Terminals:	
Motor:	max. 24 V DC, 0.7 A, via 4-mm safety sockets
Eddy current brake:	0 to 24 V DC, max. 2 A, via 4-mm safety sockets
Scale ring:	300 mm Ø
Dimensions:	400 mm x 140 mm x 270 mm
Ground:	4 kg

2.1 Scope of supply

1 Torsional pendulum
2 Additional 10 g weights
2 Additional 20 g weights

3. Theoretical Fundamentals

3.1 Symbols used in the equations

D	=	Angular directional variable
J	=	Mass moment of inertia
M	=	Restoring torque
T	=	Period
T_0	=	Period of an undamped system
T_d	=	Period of the damped system
$\widehat{M}_E$	=	Amplitude of the exciter moment
b	=	Damping torque
n	=	Frequency
t	=	Time
Λ	=	Logarithmic decrement
δ	=	Damping constant
φ	=	Angle of deflection
$\widehat{\varphi}_0$	=	Amplitude at time t = 0 s
$\widehat{\varphi}_n$	=	Amplitude after n periods
$\widehat{\varphi}_E$	=	Exciter amplitude
$\widehat{\varphi}_S$	=	System amplitude
ω_0	=	Natural frequency of the oscillating system
ω_d	=	Natural frequency of the damped system
ω_E	=	Exciter angular frequency
$\omega_{E\,res}$	=	Exciter angular frequency for max. amplitude
Ψ_{0S}	=	System zero phase angle

3.2 Harmonic rotary oscillation

A harmonic oscillation is produced when the restoring torque is proportional to the deflection. In the case of harmonic rotary oscillations the restoring torque is proportional to the deflection angle φ:

$$M = D \cdot \varphi$$

The coefficient of proportionality D (angular direction variable) can be computed by measuring the deflection angle and the deflection moment.
If the period duration T is measured, the natural resonant frequency of the system ω_0 is given by

$$\omega_0 = 2\,\pi/T$$

and the mass moment of inertia J is given by

$$\omega_0^2 = \frac{D}{J}$$

3.3 Free damped rotary oscillations

An oscillating system that suffers energy loss due to friction, without the loss of energy being compensated for by any additional external source, experiences a constant drop in amplitude, i.e. the oscillation is damped.
At the same time the damping torque b is proportional to the deflectional angle $\dot{\varphi}$.
The following motion equation is obtained for the torque at equilibrium

$$J \cdot \ddot{\varphi} + b \cdot \dot{\varphi} + D \cdot \varphi = 0$$

b = 0 for undamped oscillation.
If the oscillation begins with maximum amplitude $\widehat{\varphi}_0$ at t = 0 s the resulting solution to the differential equation for light damping ($\delta^2 < \omega_0^2$) (oscillation) is as follows

$$\varphi = \widehat{\varphi}_0 \cdot e^{-\delta \cdot t} \cdot \cos(\omega_d \cdot t)$$

$\delta = b/2\,J$ is the damping constant and

$$\omega_d = \sqrt{\omega_0^2 - \delta^2}$$

the natural frequency of the damped system.
Under heavy damping ($\delta^2 > \omega_0^2$) the system does not oscillate but moves directly into a state of rest or equilibrium (non-oscillating case).
The period duration T_d of the lightly damped oscillating system varies only slightly from T_0 of the undamped oscillating system if the damping is not excessive.
By inserting $t = n \cdot T_d$ into the equation

$$\varphi = \widehat{\varphi}_0 \cdot e^{-\delta \cdot t} \cdot \cos(\omega_d \cdot t)$$

and $\varphi = \widehat{\varphi}_n$ for the amplitude after n periods we obtain the following with the relationship $\omega_d = 2\,\pi/T_d$

$$\frac{\widehat{\varphi}_n}{\widehat{\varphi}_0} = e^{-n \cdot \delta \cdot T_d}$$

and thus from this the logarithmic decrement Λ:

$$\Lambda = \delta \cdot T_d = \frac{1}{n} \cdot ln\left[\frac{\widehat{\varphi}_n}{\widehat{\varphi}_0}\right] = ln\left[\frac{\widehat{\varphi}_n}{\widehat{\varphi}_{n+1}}\right]$$

By inserting $\delta = \Lambda / T_d$, $\omega_0 = 2\,\pi / T_0$ and $\omega_d = 2\pi / T_d$ into the equation

$$\omega_d = \sqrt{\omega_0^2 - \delta^2}$$

we obtain:

$$T_d = T_0 \cdot \sqrt{1 + \frac{\Lambda^2}{4\pi^2}}$$

whereby the period T_d can be calculated precisely provided that T_0 is known.

3.4 Forced oscillations

In the case of forced oscillations a rotating motion with sinusoidally varying torque is externally applied to the system. This exciter torque can be incorporated into the motion equation as follows:

$$J \cdot \ddot{\varphi} + b \cdot \dot{\varphi} + D \cdot \varphi = \widehat{M}_E \cdot \sin(\omega_E \cdot t)$$

After a transient or settling period the torsion pendulum oscillates in a steady state with the same angular frequency as the exciter, at the same time ω_E can still be phase displaced with respect to ω_0. Ψ_{0S} is the system's zero-phase angle, the phase displacement between the oscillating system and the exciter.

$$\varphi = \widehat{\varphi}_S \cdot \sin(\omega_E \cdot t - \Psi_{0S})$$

The following holds true for the system amplitude $\widehat{\varphi}_S$

$$\widehat{\varphi} = \frac{\frac{\widehat{M}_E}{J}}{\sqrt{(\omega_0^2 - \omega_E^2)^2 + 4\delta^2 \cdot \omega_E^2}}$$

The following holds true for the ratio of system amplitude to the exciter amplitude

$$\frac{\widehat{\varphi}_S}{\widehat{\varphi}_E} = \frac{\frac{\widehat{M}_E}{J}}{\sqrt{\left[1 - \left(\frac{\omega_E}{\omega_0}\right)^2\right]^2 + 4\left(\frac{\delta}{\omega_0}\right)^2 \cdot \left(\frac{\omega_E}{\omega_0}\right)^2}}$$

In the case of undamped oscillations, theoretically speaking the amplitude for resonance (ω_E equal to ω_0) increases infinitely and can lead to "catastrophic resonance".

In the case of damped oscillations with light damping the system amplitude reaches a maximum where the exciter's angular frequency $\omega_{E\,res}$ is lower than the system's natural frequency. This frequency is given by

$$\omega_{Eres} = \omega_0 \cdot \sqrt{1 - \frac{2\delta^2}{\omega_0^2}}$$

Stronger damping does not result in excessive amplitude.

For the system's zero phase angle Ψ_{0S} the following is true:

$$\Psi_{0S} = \arctan\left(\frac{2\delta\,\omega}{\omega_0^2 - \omega_\omega^2}\right)$$

For $\omega_E = \omega_0$ (resonance case) the system's zero-phase angle is $\Psi_{0S} = 90°$. This is also true for $\delta = 0$ and the oscillation passes its limit at this value.

In the case of damped oscillations ($\delta > 0$) where $\omega_E < \omega_0$, we find that $0° \leq \Psi_{0S} \leq 90°$ and when $\omega_E > \omega_0$ it is found that $90° \leq \Psi_{0S} \leq 180°$.

In the case of undamped oscillations ($\delta = 0$), $\Psi_{0S} = 0°$ for $\omega_E < \omega_0$ and $\Psi_{0S} = 180°$ for $\omega_E > \omega_0$.

4. Operation

4.1 Free damped rotary oscillations

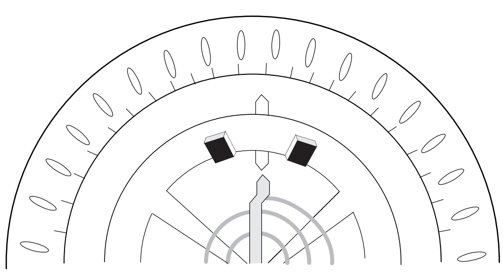
- Connect the eddy current brake to the variable voltage output of the DC power supply for torsion pendulum.
- Connect the ammeter into the circuit.
- Determine the damping constant as a function of the current.

4.2 Forced oscillations

- Connect the fixed voltage output of the DC power supply for the torsion pendulum to the sockets (16) of the exciter motor.
- Connect the voltmeter to the sockets (15) of the exciter motor.
- Determine the oscillation amplitude as a function of the exciter frequency and of the supply voltage.
- If needed connect the eddy current brake to the variable voltage output of the DC power supply for the torsion pendulum.

4.3 Chaotic oscillations

- To generate chaotic oscillations there are 4 supplementary weights at your disposal which alter the torsion pendulum's linear restoring torque.
- To do this screw the supplementary weight to the body of the pendulum (5).



5. Example experiments

5.1 Free damped rotary oscillations

- To determine the logarithmic decrement Λ, the amplitudes are measured and averaged out over several runs. To do this the left and right deflections of the torsional pendulum are read off the scale in two sequences of measurements.
- The starting point of the pendulum body is located at +15 or –15 on the scale. Take the readings for five deflections.
- From the ratio of the amplitudes we obtain Λ using the following equation

$$\Lambda = ln\left[\frac{\widehat{\varphi}_n}{\widehat{\varphi}_{n+1}}\right]$$

n	$\widehat{\varphi}$ –				$\widehat{\varphi}$ +			
0	–15	–15	–15	–15	15	15	15	15
1	–14.8	–14.8	–14.8	–14.8	14.8	14.8	14.8	14.8
2	–14.4	–14.6	–14.4	–14.6	14.4	14.4	14.6	14.4
3	–14.2	–14.4	–14.0	–14.2	14.0	14.2	14.2	14.0
4	–13.8	–14.0	–13.6	–14.0	13.8	13.8	14.0	13.8
5	–13.6	–13.8	–13.4	–13.6	13.4	13.4	13.6	13.6

n	Ø $\widehat{\varphi}$ –	Ø $\widehat{\varphi}$ +	Λ –	Λ +
0	–15	15		
1	–14.8	14.8	0.013	0.013
2	–14.5	14.5	0.02	0.02
3	–14.2	14.1	0.021	0.028
4	–13.8	13.8	0.028	0.022
5	–13.6	13.5	0.015	0.022

- The average value for Λ comes to Λ = 0.0202.
- For the pendulum oscillation period T the following is true: $t = n \cdot T$. To measure this, record the time for 10 oscillations using a stop watch and calculate T.

  $T = 1.9$ s
- From these values the damping constant δ can be determined from δ = Λ / T.

  $\delta = 0.0106\ s^{-1}$
- For the natural frequency ω the following holds true

$$\omega = \sqrt{\left(\frac{2\pi}{T}\right)^2 - \delta^2}$$

  $\omega = 3.307$ Hz

5.2 Free damped rotary oscillations

- To determine the damping constant δ as a function of the current I flowing through the electromagnets the same experiment is conducted with an eddy current brake connected at currents of I = 0.2 A, 0.4 A and 0.6 A.

I = 0.2 A

n	$\widehat{\varphi}$ –				Ø $\widehat{\varphi}$ –	Λ –
0	–15	–15	–15	–15	–15	
1	–13.6	–13.8	–13.8	–13.6	–13.7	0.0906
2	–12.6	–12.8	–12.6	–12.4	–12.6	0.13
3	–11.4	–11.8	–11.6	–11.4	–11.5	0.0913
4	–10.4	–10.6	–10.4	–10.4	–10.5	0.0909
5	9.2	–9.6	–9.6	–9.6	–9.5	0.1

- For T = 1.9 s and the average value of Λ = 0.1006 we obtain the damping constant: δ = 0.053 s^{-1}

I = 0.4 A

n	$\widehat{\varphi}$ –				Ø $\widehat{\varphi}$ –	Λ –
0	–15	–15	–15	–15	–15	
1	–11.8	–11.8	–11.6	–11.6	–11.7	0.248
2	–9.2	–9.0	–9.0	–9.2	–9.1	0.25
3	–7.2	–7.2	–7.0	–7.0	–7.1	0.248
4	–5.8	–5.6	–5.4	–5.2	–5.5	0.25
5	–4.2	–4.2	–4.0	–4.0	–4.1	0.29

- For T = 1.9 s and an average value of Λ = 0.257 we obtain the damping constant: δ = 0.135 s^{-1}

I = 0.6 A

n	$\widehat{\varphi}$ –				Ø $\widehat{\varphi}$ –	Λ –
0	–15	–15	–15	–15	–15	
1	–9.2	–9.4	–9.2	–9.2	–9.3	0.478
2	–5.4	–5.2	–5.6	–5.8	–5.5	0.525
3	–3.2	–3.2	–3.2	–3.4	–3.3	0.51
4	–1.6	–1.8	–1.8	–1.8	–1.8	0.606
5	–0.8	–0.8	–0.8	–0.8	–0.8	0.81

- For T = 1.9 s and an average value of Λ = 0.5858 we obtain the damping constant: δ = 0.308 s^{-1}

5.3 Forced rotary oscillation

- Take a reading of the maximum deflection of the pendulum body to determine the oscillation amplitude as a function of the exciter frequency or the supply voltage.

T = 1.9 s

Motor voltage V	$\widehat{\varphi}$
3	0.8
4	1.1
5	1.2
6	1.6
7	3.3
7.6	20.0
8	16.8
9	1.6
10	1.1

- After measuring the period T the natural frequency of the system ω_0 can be obtained from

  $\omega_0 = 2\,\pi/T = 3.3069$ Hz
- The most extreme deflection arises at a motor voltage of 7.6 V, i.e. the resonance case occurs.
- Then the same experiment is performed with an eddy current brake connected at currents of I = 0.2 A, 0.4 A and 0.6 A.

I = 0.2 A

Motor voltage V	$\widehat{\varphi}$
3.0	0.9
4.0	1.1
5.0	1.2
6.0	1.7
7.0	2.9
7.6	15.2
8.0	4.3
9.0	1.8
10.0	1.1

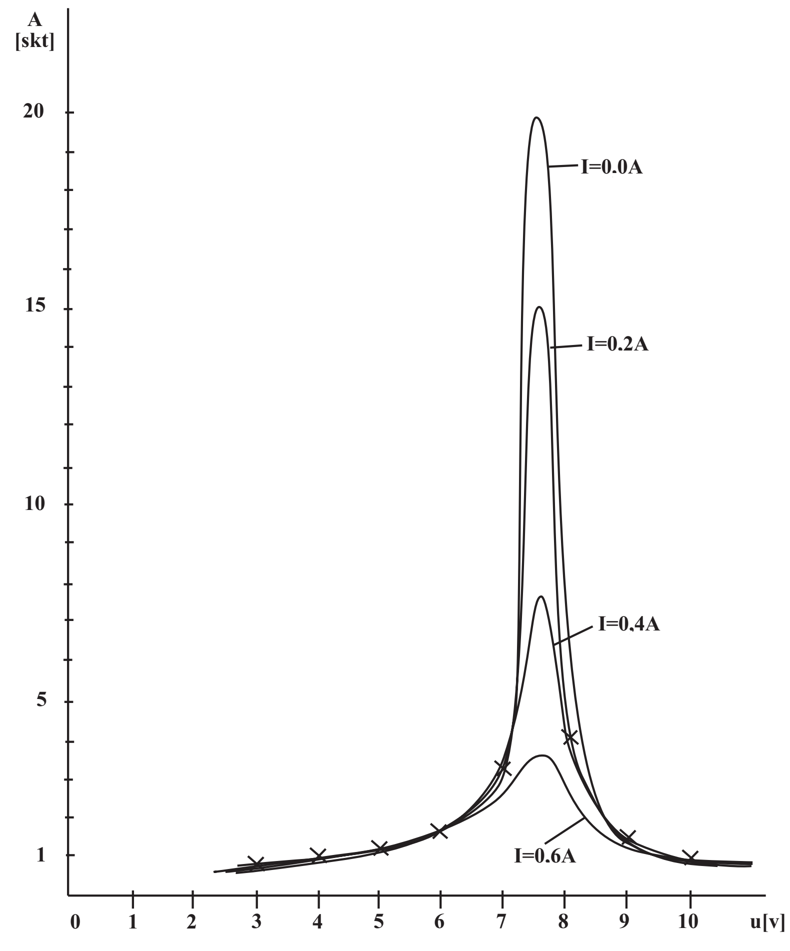
I = 0.4 A

Motor voltage V	$\widehat{\varphi}$
3.0	0.9
4.0	1.1
5.0	1.3
6.0	1.8
7.0	3.6
7.6	7.4
8.0	3.6
9.0	1.6
10.0	1.0

I = 0.6 A

Motor voltage V	$\widehat{\varphi}$
3.0	0.9
4.0	1.1
5.0	1.2
6.0	1.6
7.0	2.8
7.6.0	3.6
8.0	2.6
9.0	1.3
10.0	1.0

- From these measurements the resonance curves can be plotted in a graph depicting the amplitudes against the motor voltage.
- The resonant frequency can be determined by finding the half-width value from the graph.



Resonance curves